Spiral plat avec une seule courbe terminale externe

Anisochronisme en position horizontale

Approximations de Haag et angle d'enroulement optimum

Caractéristiques du spiral

Référence :C:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :C:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L := L_{sp} \quad L = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56\times 10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$$a := \frac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_A - a\cdot(\alpha-\alpha_A) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$$

$$s(\alpha) := \frac{1}{2\cdot a}\cdot\left(r_A{}^2 - r_s(\alpha)^2\right) \qquad s(\alpha) := r_A\cdot(\alpha-\alpha_A) - \frac{a}{2}\cdot(\alpha-\alpha_A)^2$$

Moment quadratique de section

Référence :C:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Courbe terminale externe

$$r_{t1} := 0.8 \quad r_{t1} := racine\left[(2\cdot r_{t1}-1)^4 - 4\cdot(1-r_{t1})^4 - \pi^2\cdot r_{t1}{}^2\cdot(1-r_{t1})^2, r_{t1}\right]\cdot r_A \quad r_{t1} = 0.832\,r_A$$

$$r_{t2} := 2\cdot r_{t1} - r_A \quad r_{t2} = 0.665\,r_A \quad \beta_0 := arctan\left[\frac{\pi\cdot r_{t1}}{2\cdot(r_A - r_{t1})}\right] \quad \beta_0 = 82.695\,deg \quad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$$

$$X_{0t1}(\alpha_t) := r_A - r_{t1} + r_{t1}\cdot cos(\alpha_t) \quad Y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t) \quad X_{0t2}(\beta_t) := -r_{t2}\cdot cos(\beta_t) \quad Y_{0t2}(\beta_t) := -r_{t2}\cdot sin(\beta_t)$$

Paramètres de la courbe terminale externe

$$X_1 := \frac{1}{r_A{}^2}\cdot\left(\int_0^{\pi} X_{0t1}(\alpha)\cdot r_{t1}\,d\alpha + \int_0^{\beta_0} X_{0t2}(\beta)\cdot r_{t2}\,d\beta\right) \qquad X_1 = 0$$

$$Y_1 := \frac{1}{r_A{}^2}\cdot\left(\int_0^{\pi} Y_{0t1}(\alpha)\cdot r_{t1}\,d\alpha + \int_0^{\beta_0} Y_{0t2}(\beta)\cdot r_{t2}\,d\beta\right) - 1 \qquad Y_1 = 0$$

$$\rho_1 := \sqrt{X_1{}^2 + Y_1{}^2} \quad \varphi_1 := Atan(X_1, Y_1) \quad \boxed{\rho_1 = 0} \quad \boxed{\varphi_1 = 270\,deg}$$

$$X_2 := \frac{1}{r_A{}^3}\cdot\left[\int_0^{\pi} r_{t1}\cdot\alpha\cdot X_{0t1}(\alpha)\cdot r_{t1}\,d\alpha + \int_0^{\beta_0} (r_{t1}\cdot\pi + r_{t2}\cdot\beta)\cdot X_{0t2}(\beta)\cdot r_{t2}\,d\beta\right] + 1$$

$$Y_2 := \frac{1}{r_A^{\,3}} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot Y_{0t1}(\alpha) \cdot r_{t1}\, d\alpha + \int_0^{\beta_0} \left(r_{t1} \cdot \pi + r_{t2} \cdot \beta \right) \cdot Y_{0t2}(\beta) \cdot r_{t2}\, d\beta \right]$$

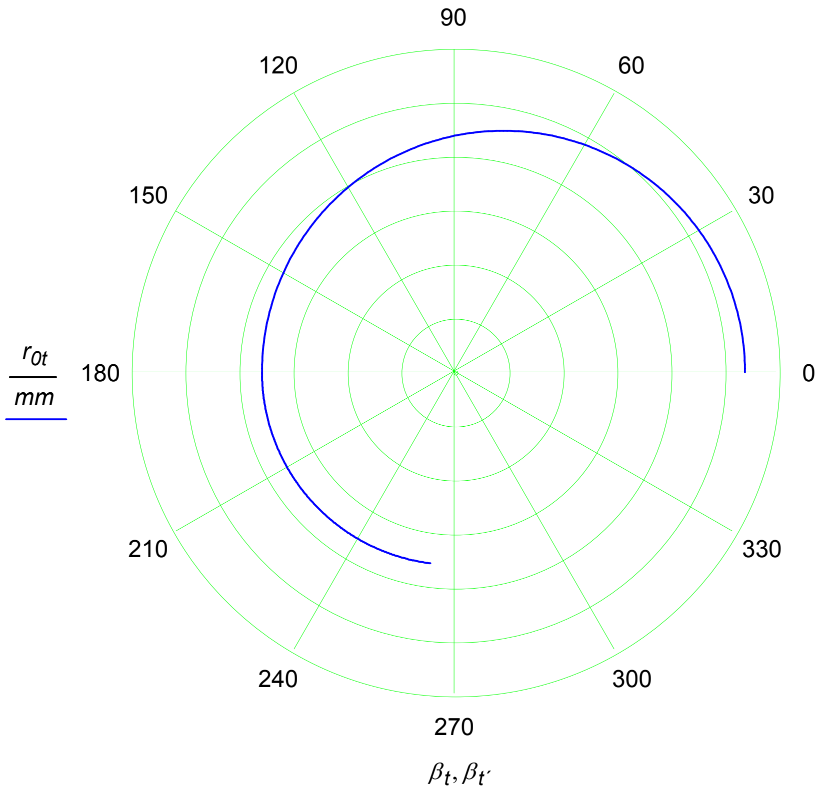
$\rho_2 := \sqrt{X_2^{\,2} + Y_2^{\,2}}$ $\qquad \varphi_2 := Atan\left(X_2, Y_2\right)$ $\qquad \rho_2 = 1.055$ $\qquad \varphi_2 = 147.579\, deg$

Graphe

$n_t := 201$ $\quad j := 0 .. n_t - 1$ $\quad \Delta\alpha_t := \dfrac{\pi}{n_t - 1}$ $\quad \alpha_{t_j} := j \cdot \Delta\alpha_t$ $\quad X_{t_j} := X_{0t1}\left(\alpha_{t_j}\right)$ $\quad Y_{t_j} := Y_{0t1}\left(\alpha_{t_j}\right)$

$\Delta\beta_t := \dfrac{\beta_0}{n_t - 1}$ $\quad \beta_{t_j} := j \cdot \Delta\beta_t$ $\quad X_{t2_j} := X_{0t2}\left(\beta_{t_j}\right)$ $\quad Y_{t2_j} := Y_{0t2}\left(\beta_{t_j}\right)$ $\quad X_t := pile\left(X_t, X_{t2}\right)$ $\quad Y_t := pile\left(Y_t, Y_{t2}\right)$

$r_{0t} := \overrightarrow{\sqrt{X_t^{\,2} + Y_t^{\,2}}}$ $\qquad \beta_t := \overrightarrow{Atan\left(X_t, Y_t\right)}$



Déplacement de la virole libre

$\boldsymbol{OA} := r_A \cdot e^{i \cdot \pi}$ $\qquad r_B := 0.5 \cdot d1_{sp}$ $\qquad \boldsymbol{OB} := r_B \cdot e^{i \cdot \left(\pi + \psi_0\right)}$ $\qquad L_t := l_t + L$

$$\boldsymbol{w_A}\left(\rho_1, \theta\right) := \frac{\theta}{L_t} \cdot \left[i \cdot \left(r_A \cdot \rho_1 \cdot e^{-i \cdot \varphi_1} + 2 \cdot a \right) + \frac{\theta}{L_t} \cdot r_A^{\,2} \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \right] \cdot exp\left(i \cdot \theta \cdot \frac{l_t}{L_t} \right) \cdot \boldsymbol{OA}$$

$$\boldsymbol{w_B}(\theta) := \frac{\theta}{L_t} \cdot \left[-i \cdot \left(i \cdot r_B + 2 \cdot a \right) - \frac{\theta}{L_t} \cdot r_B^{\,2} \right] \cdot exp\left(i \cdot \theta \cdot \frac{l_t + L}{L_t} \right) \cdot \boldsymbol{OB}$$

$\boldsymbol{w}\left(\rho_1, \theta\right) := \boldsymbol{w_A}\left(\rho_1, \theta\right) + \boldsymbol{w_B}(\theta)$ $\qquad \boldsymbol{w}\left(\rho_1, \theta_0\right) = 0.023 + 4.295i \times 10^{-3}\, mm$

En éliminant les termes de second ordre

$$\mathbf{w_{ah}}(\theta) := \frac{\theta}{L_t}\cdot\left[i\cdot\left(r_A\cdot\rho_1\cdot e^{-i\cdot\varphi_1} + 2\cdot a\right) + \frac{\theta}{L_t}\cdot r_A^{\ 2}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right]\cdot\mathbf{OA} + \frac{\theta}{L_t}\cdot\left[-i\cdot\left(i\cdot r_B + 2\cdot a\right) - \frac{\theta}{L_t}\cdot r_B^{\ 2}\right]\cdot\mathbf{OB}\cdot e^{i\cdot\theta}$$

$$\mathbf{w_{ah}}(\theta_0) = 0.025 - 3.503i \times 10^{-4}\,mm$$

Réaction sur le pivot de balancier

$$\sigma 2 := \frac{r_A^{\ 2} + r_B^{\ 2}}{2} \qquad \sigma 2 = 2.705\,mm^2 \qquad \mathbf{F}(\theta) := 2\cdot\frac{E\cdot I_{33}}{L\cdot\sigma 2}\cdot\mathbf{w_{ah}}(\theta) \qquad \left|\mathbf{F}(\theta_0)\right| = 1.191\times 10^{-5}\,N$$

Perturbation de période

$$X_w(\theta) := \frac{\left(\left|\mathbf{w}(\rho_1,\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_w(\theta) := \frac{d}{d\theta}X_w(\theta) \qquad \delta_w(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_w(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$$

$$\mu_w(\theta_0) := -86400\cdot\delta_w(\theta_0) \qquad \boxed{\mu_w(\theta_0) = 1.359} \qquad \boxed{\mu_w(180\cdot deg) = 0.769}$$

$$X_{ah}(\theta) := \frac{\left(\left|\mathbf{w_{ah}}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_{ah}(\theta) := \frac{d}{d\theta}X_{ah}(\theta) \qquad \delta_{ah}(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_{ah}(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = 1.251} \qquad \boxed{\mu_{ah}(180\cdot deg) = 0.816}$$

$$A := \frac{1}{L_t^{\ 2}}\cdot\left[r_A^{\ 4}\cdot\rho_1^{\ 2} + r_B^{\ 4} + 4\cdot a\cdot r_A^{\ 3}\cdot\rho_1\cdot\cos(\varphi_1) + 4\cdot a^2\cdot\left(r_A^{\ 2} + r_B^{\ 2}\right)\right]$$

$$B := \frac{3}{2\cdot L_t^{\ 4}}\cdot\left(r_A^{\ 6}\cdot\rho_2^{\ 2} + r_B^{\ 6}\right)$$

$$C := \frac{2}{L_t^{\ 2}}\cdot\left[r_A^{\ 2}\cdot r_B^{\ 2}\cdot\rho_1\cdot\sin(\psi_0 + \varphi_1) + 2\cdot a\cdot r_A\cdot r_B\cdot\left(r_B\cdot\sin(\psi_0) - r_A\cdot\rho_1\cdot\cos(\psi_0 + \varphi_1) - 2\cdot a\cdot\cos(\psi_0)\right)\right]$$

$$D1 := r_A\cdot r_B^{\ 2}\cdot\rho_1\cdot\cos(\psi_0 + \varphi_1) + r_A^{\ 2}\cdot r_B\cdot\rho_2\cdot\sin(\psi_0 + \varphi_2)$$

$$D := \frac{2\cdot r_A\cdot r_B}{L_t^{\ 3}}\cdot\left[D1 + 2\cdot a\cdot\left(r_B^{\ 2}\cdot\cos(\psi_0) - r_A^{\ 2}\cdot\rho_2\cdot\cos(\psi_0 + \varphi_2)\right)\right]$$

$$K := \frac{2}{L_t^{\ 4}}\cdot r_A^{\ 3}\cdot r_B^{\ 3}\cdot\rho_2\cdot\cos(\psi_0 + \varphi_2)$$

$$F(x) := J0(x) - x\cdot J1(x) \qquad H(x) := x\cdot\left(1 + x^2\right)\cdot J1(x) - 2\cdot x^2\cdot J0(x) \qquad G(x) := -x\cdot\left(J1(x) + x\cdot J0(x)\right)$$

$$\delta_{ah}(\theta_0) := \frac{-1}{\sigma 2}\cdot\left(A + B\cdot\theta_0^{\ 2} + C\cdot F(\theta_0) + D\cdot G(\theta_0) + K\cdot H(\theta_0)\right)$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = 1.251} \qquad \boxed{\mu_{ah}(180\cdot deg) = 0.816}$$

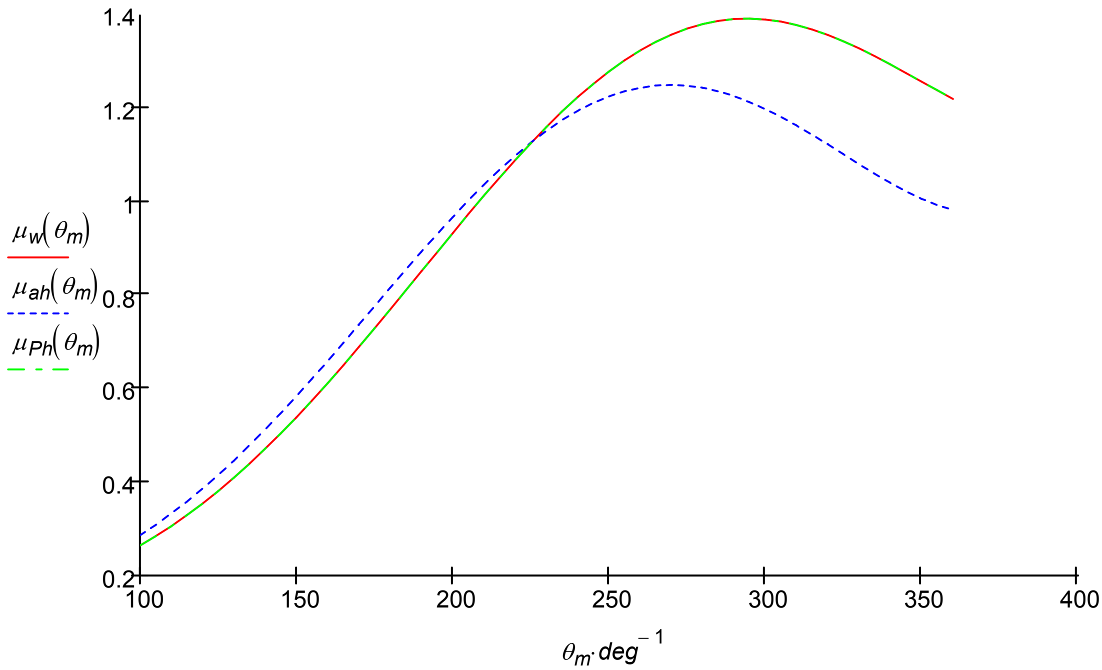
Spiral muni de courbes Phillips

$\boldsymbol{w_{Ph}}(\theta) := \boldsymbol{w}(0,\theta)$ $\qquad$ $\boldsymbol{w_{Ph}}(\theta_0) = 0.023 + 4.295i \times 10^{-3}\,mm$

$$X_{Ph}(\theta) := \frac{\left(\left|\boldsymbol{w_{Ph}}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_{Ph}(\theta) := \frac{d}{d\theta}X_{Ph}(\theta) \qquad \delta_{Ph}(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_{Ph}(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$$

$\mu_{Ph}(\theta_0) := -86400\cdot\delta_{Ph}(\theta_0)$ $\qquad$ $\boxed{\mu_{Ph}(\theta_0) = 1.359}$ $\qquad$ $\boxed{\mu_{Ph}(180\cdot deg) = 0.769}$

$\theta_m := 100\cdot deg, 105\cdot deg .. 360\cdot deg$



Angle d'enroulement optimum

$$L(\psi_0) := r_A\cdot\psi_0 - \frac{1}{2}\cdot a\cdot\psi_0^{\,2} \qquad L_t(\psi_0) := L(\psi_0) + l_t \qquad r_B(\psi_0) := r_A - a\cdot\psi_0 \qquad \sigma 2(\psi_0) := \frac{r_A^{\,2} + r_B(\psi_0)^2}{2}$$

$$A(\psi_0) := \frac{1}{L_t(\psi_0)^2}\cdot\left[r_B(\psi_0)^4 + 4\cdot a^2\cdot\left(r_A^{\,2} + r_B(\psi_0)^2\right)\right] \qquad B(\psi_0) := \frac{3}{2\cdot L_t(\psi_0)^4}\cdot\left(r_A^{\,6}\cdot\rho_2^{\,2} + r_B(\psi_0)^6\right)$$

$$C(\psi_0) := \frac{2}{L_t(\psi_0)^2}\cdot\left[2\cdot a\cdot r_A\cdot r_B(\psi_0)\cdot\left(r_B(\psi_0)\cdot\sin(\psi_0) - 2\cdot a\cdot\cos(\psi_0)\right)\right]$$

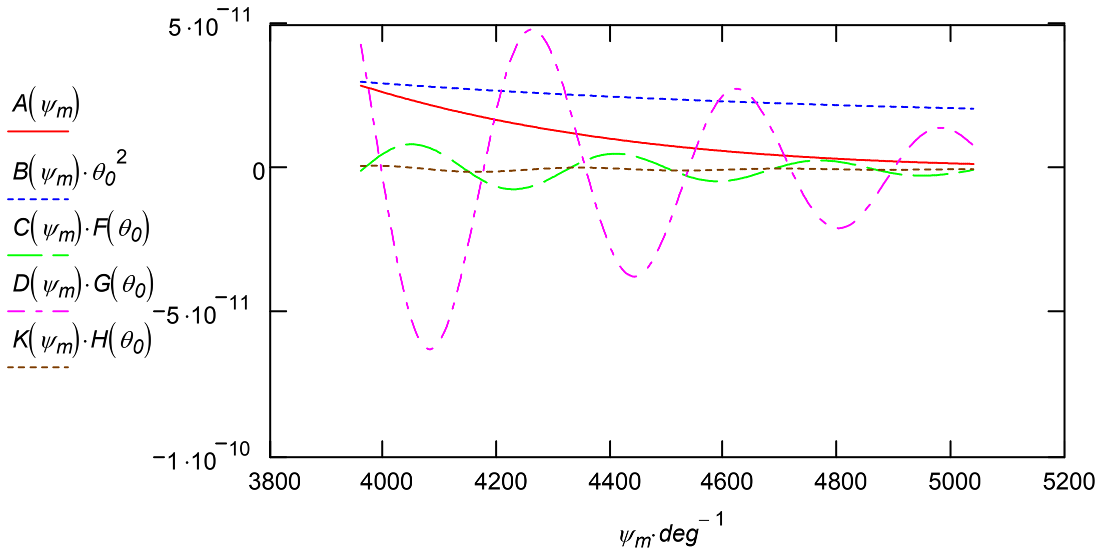
$$D(\psi_0) := \frac{2\cdot r_A\cdot r_B(\psi_0)}{L_t(\psi_0)^3}\cdot\left[r_A^{\,2}\cdot r_B(\psi_0)\cdot\rho_2\cdot\sin(\psi_0+\varphi_2) + 2\cdot a\cdot\left(r_B(\psi_0)^2\cdot\cos(\psi_0) - r_A^{\,2}\cdot\rho_2\cdot\cos(\psi_0+\varphi_2)\right)\right]$$

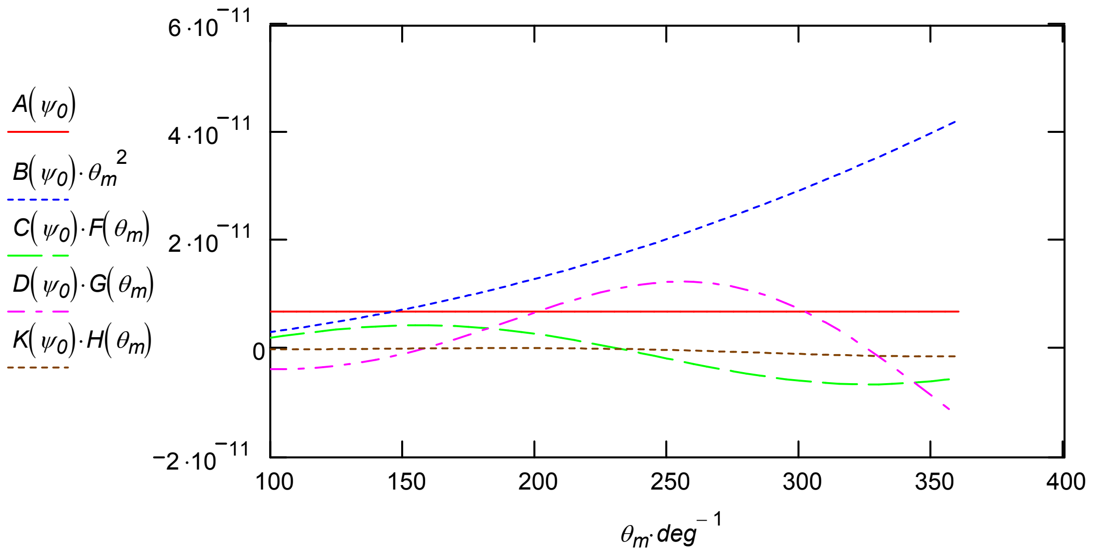
$$K(\psi_0) := \frac{2}{L_t(\psi_0)^4}\cdot r_A^{\,3}\cdot r_B(\psi_0)^3\cdot\rho_2\cdot\cos(\psi_0+\varphi_2)$$

$$\delta_{a\psi}(\psi_0,\theta_0) := \frac{-1}{\sigma 2(\psi_0)}\cdot\left(A(\psi_0) + B(\psi_0)\cdot\theta_0^{\,2} + C(\psi_0)\cdot F(\theta_0) + D(\psi_0)\cdot G(\theta_0) + K(\psi_0)\cdot H(\theta_0)\right)$$

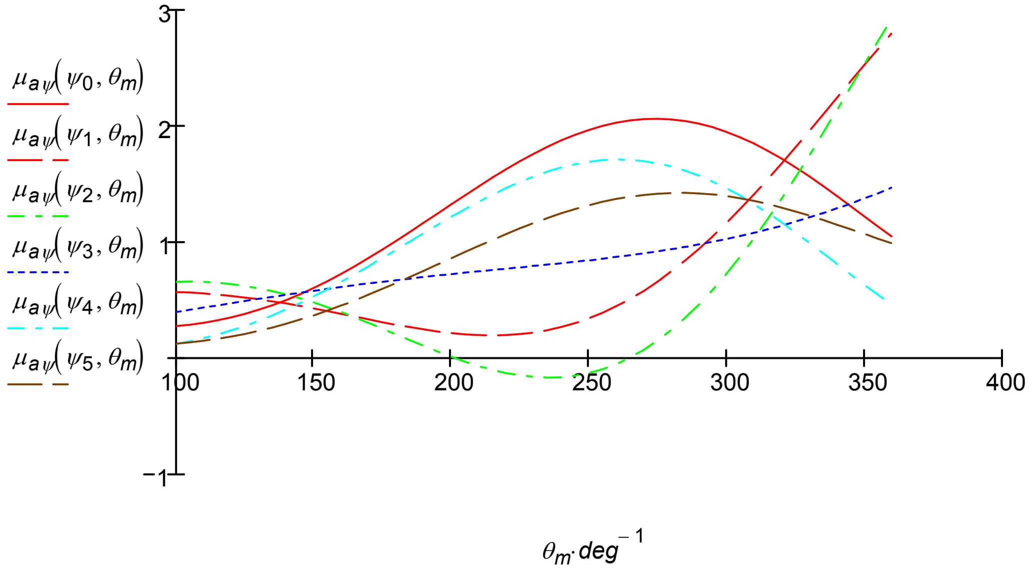
$\mu_{a\psi}(\psi_0,\theta_0) := -86400\cdot\delta_{a\psi}(\psi_0,\theta_0)$ $\qquad$ $\boxed{\mu_{a\psi}(\psi_0,\theta_0) = 1.251}$ $\qquad$ $\boxed{\mu_{a\psi}(\psi_0, 180\cdot deg) = 0.816}$

$\psi_m := 2\cdot\pi\cdot 11, 2\cdot\pi\cdot 11.01 .. 2\cdot\pi\cdot 14$





$ns := 0 .. 5$ $\qquad \psi_{ns} := 2\cdot\pi\cdot\left(12 + \frac{ns}{5}\right)$ $\qquad \frac{1}{2\cdot\pi}\cdot\psi^T = (12 \quad 12.2 \quad 12.4 \quad 12.6 \quad 12.8 \quad 13)$



$$\mathbf{w_A}(\rho_1, \theta, \psi_0) := \frac{\theta}{L_t(\psi_0)} \cdot \left[i \cdot \left(r_A \cdot \rho_1 \cdot e^{-i \cdot \varphi_1} + 2 \cdot a \right) + \frac{\theta}{L_t(\psi_0)} \cdot r_A{}^2 \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \right] \cdot exp\left(i \cdot \theta \cdot \frac{l_t}{L_t(\psi_0)} \right) \cdot \left(r_A \cdot e^{i \cdot \pi} \right)$$

$$\mathbf{w_B}(\theta, \psi_0) := \frac{\theta}{L_t(\psi_0)} \cdot \left[-i \cdot \left(i \cdot r_B(\psi_0) + 2 \cdot a \right) - \frac{\theta}{L_t(\psi_0)} \cdot r_B(\psi_0)^2 \right] \cdot exp\left(i \cdot \theta \cdot \frac{l_t + L(\psi_0)}{L_t(\psi_0)} \right) \cdot \left[r_B(\psi_0) \cdot e^{i \cdot (\pi + \psi_0)} \right]$$

$$\mathbf{w}(\rho_1, \theta, \psi_0) := \mathbf{w_A}(\rho_1, \theta, \psi_0) + \mathbf{w_B}(\theta, \psi_0) \qquad \mathbf{w}(\rho_1, \theta_0, \psi_0) = 0.023 + 4.295i \times 10^{-3}\, mm$$

$$X_w(\rho_1, \theta, \psi_0) := \frac{\left(\left| \mathbf{w}(\rho_1, \theta, \psi_0) \right| \right)^2}{\sigma2(\psi_0)} \qquad \gamma_w(\rho_1, \theta, \psi_0) := \frac{d}{d\theta} X_w(\rho_1, \theta, \psi_0)$$

$$\delta_w(\rho_1, \theta_0, \psi_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2 \cdot \pi} \gamma_w(\rho_1, \theta_0 \cdot cos(\varphi), \psi_0) \cdot cos(\varphi)\, d\varphi$$

$$\mu_w(\rho_1, \theta_0, \psi_0) := -86400 \cdot \delta_w(\rho_1, \theta_0, \psi_0) \qquad \boxed{\mu_w(\rho_1, \theta_0, \psi_0) = 1.359} \qquad \boxed{\mu_w(\rho_1, 180 \cdot deg, \psi_0) = 0.769}$$